

STUDY OF PLASMA RESPONSE TO 3D EXTERNAL MAGNETIC FIELD PERTURBATIONS

Marco Neri^{1,2,3}, Raffaele Albanese^{1,2}, Antonio Iaiunese⁴, Pasquale Zumbolo^{2,5}

¹DIETI, Università degli studi di Napoli Federico II, 80125 Napoli

²Consorzio CREATE, via Claudio 21, I-80124 Napoli

³Università degli studi di Padova, Padova, Via Gradenigo 6/b, I-35131 Padova

⁴ENEA, Via E. Fermi 45, Frascati, 00044 Rome, Italy

⁵EUROfusion – Programme Management Unit, Garching, Germany

Accurate modelling of magnetic field in tokamak devices is a relevant topic when the plasma equilibrium stability must be analyzed. For this purpose, linearized models are successfully involved both in the design phases of the plasma scenario and control system. All the approaches here proposed stand on the equations of the single-fluid MHD model, i.e., by treating the plasma as a fluid conductor crossed by current density.

Under the assumption that both plasma and external structures are axisymmetric, the plasma reaction to external magnetic perturbations can be derived from a Finite Element Method (FEM) fixed boundary plasma equilibrium solution. This is the procedure adopted in the CREATE-FXL code [1]. The linearized variation of the plasma equilibrium quantities (such as the total poloidal flux per radian ψ , and plasma flux contribution ψ_{pl}) is directly evaluated by perturbing the degrees of freedom (*DoFs*) of the model, which are the equilibrium total poloidal flux values $\psi_{b,ib}$, $ib = 1, \dots, N_b$ at the plasma boundary mesh nodes ib , and the vector of shape parameters α defining the toroidal plasma current density function $J_\phi(r, \psi, \alpha)$. In this model, J_ϕ is written in terms of nodal quantities, as described in [2], to treat also non-regular profile features, like cusps, or discontinuities. Then, using a functional approach based on Gateaux derivatives, the perturbed toroidal current density δJ_ϕ , and the consequent plasma contribution $\delta \psi_{pl}$, are computed in the assigned plasma region. Finally, all the variations are related to the perturbations $\delta \psi_{ext,b}$ of the poloidal flux per radian due to the external sources at the plasma boundary nodes. In this way, the plasma boundary represents the coupling surface between plasma and external sources. Using a consequence of the well-known reciprocity theorem in circuits theory, also the plasma poloidal flux linked to the external circuits is returned. Therefore, the main outputs of the model are matrices of a *RL* circuit equation modelling the evolution of the plasma-tokamak system near equilibrium:

$$L \frac{dI}{dt} + RI = U$$

The inductance matrix among plasma and surrounding conductors is not symmetric due to the presence of plasma; its eigenvalues clearly mark the system instability. The described linearization technique is advantageous in terms of computational effort since only the plasma region must be meshed. Furthermore, thanks to the nodal formulation for J_ϕ , FXL has proved to be a valuable tool in a high range of current density profiles. The code has been intensely used in various unstable vertical analyses, returning the evolution growth rate and the stability margins.

The features of CREATE-FXL justify its extension also to 3D non-axisymmetric external structures, which are expected to be in future tokamak devices [3]. In this case, the coupling

surface remains the plasma boundary, and the only difference from the previous procedure is that the external potential vector field perturbation $\delta \mathbf{A}_{\phi, ext}(r, \phi, z)$ must be averaged along the toroidal direction.

Both the previous models can be then compared with a fully 3D linearized ideal MHD model, described by the following 2nd order linear PDE [4]:

$$\mathbf{F}(\xi) = \rho \ddot{\xi}$$

Where F is called *force operator*. Assuming that $\xi = Re(\xi_0 e^{i\omega t})$, the problem is written in weak formulation and then solved using the FEM method with piecewise linear vectorial basis functions $\mathbf{w}_k(r, z)$ in the R-Z plane, and harmonic functions in the toroidal direction ϕ . The stiffness matrix K corresponding to F is fast assembled in a vectorial way, using the technique described in [5]. Then, some simple analytical tests are carried out to validate the procedure. One is to impose a rigid plasma boundary vertical displacement ($n=0$). Both the cases with $\rho = 0$ and $\rho \neq 0$ are investigated; indeed, in the first case, the solution is not unique but at least the trivial one (vertical displacement of all plasma region) is analytically known, whereas for the second case, no analytical solution is known to us. From ξ , the perturbed total magnetic field $d\mathbf{B} = \nabla \times (\xi \times \mathbf{B}_{eq})$, where $\mathbf{B}_{eq}$ is the equilibrium field, is evaluated.

Consequently, the 3D current density variation $\delta \mathbf{J}$ can be evaluated. Then, by using the same approach of CREATE-FXL, the plasma quantities inside the plasma region are related to the components of the magnetic field perturbations $\delta \mathbf{B}_{ext, b}$ at the plasma boundary due to 3D external sources. The model can then be used to study the deformation due to non-axisymmetric sources, e.g., the toroidal field ripple, error fields plasma response, and the growth rate of the resistive wall modes.

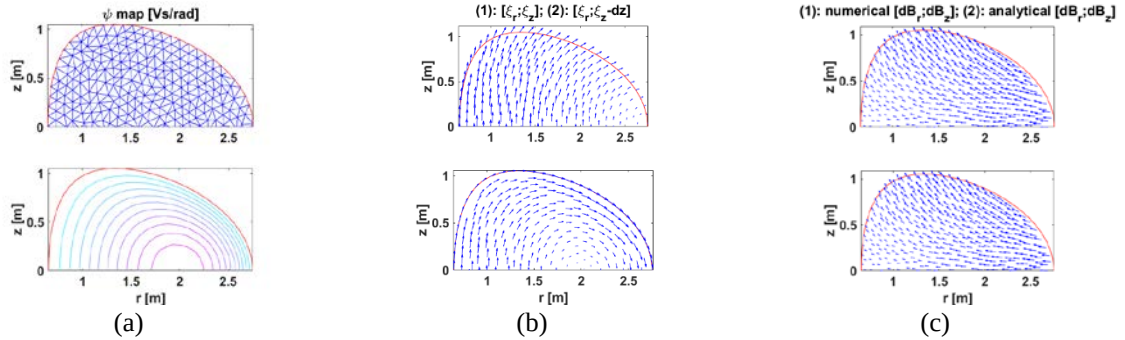


Fig.1. (a) Solovlev plasma mesh and equilibrium; (b) numerical ξ arrows, and $\xi - \xi_{vert}$ at the mesh nodes with $\rho = 0$; (c) numerical and analytical $d\mathbf{B}$ arrows

References

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